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Global optimization and complementarity for solving a semi-actuated traffic control problem

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Abstract

The aim of this paper is to find the optimal green split allocation for a queuing system. This system results from a signalized intersection regulated by semi-actuated control in an urban traffic network. The model in question has been formulated as a Mathematical Program with Equilibrium (or Complementarity) Constraints (MPEC). Computational experiments with a sequential complementarity algorithm to attain a global minimum for the MPEC are included to estimate the green times and cycle lengths.

Keywords: Traffic Management; Semi-actuated Control; Complementarity; Global Optimization

1. Introduction

Solving the problem of severe traffic congestion has risen to the top of the agenda in many cities. Traffic signals operating with semi-actuated control have been widely used on secondary streets, since they provide flexible controls adjustable by traffic volumes, reducing vehicle's delay.

The actuated control is a strategy of response to important variations in the traffic conditions. The main problem when using semi-actuated control is the difficulty in selecting the best combination of maximum, minimum and unit extension of green time. On the other hand, queuing theory is a widely used alternative tool to compute performance measures for traffic signals, where we have costumers (vehicles) arriving at a service point (intersection regulated by traffic control signals). It is well known that vehicles only leave the intersection during the green period and the signal phases in semi-actuated control are not of fixed length. This does not foster application of the queuing theory which is restricted to rather simpler traffic conditions. Akçelik (1994) and Lin (1990) proposed analytical methods for estimating average green times and average cycle lengths for actuated signals.

According to the literature semi-actuated signal operations are being used separately or as part of a system of coordinated traffic signals. The study of the influence of the characteristics of the arrivals and departures of traffic in the performance of signalized intersections with this type of control started shortly after the actuated control has

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been first used (Akcelik 1994). The need to optimize the parameters of the controller and the position of the detector as well as to investigate the relationship between these factors has appeared as a motivation for different studies (Lin 1991; Rouphail et al. 1997).

Semi-actuated control is a traffic management resource deployed at intersections where a main street intersects a secondary street, a sensor being installed along the secondary street. The main street always gets the green signal for at least a fixed minimum green time during a signal cycle. If the sensor is activated during this interval the main street remains on green until the minimum green time is reached. The green signal is then given to the secondary street until traffic is cleared or until green time reaches a fixed maximum (whichever occurs first); then the green signal is transferred back to the main street. If no vehicle is detected along the minor approach the period of green in the main street is prolonged until a vehicle is detected by the sensor located on the secondary street. This procedure is illustrated in Figure 1.

An actuated signal is assumed to be extremely efficient in the use of available green time. The efficient use of semi-actuated control requires careful selection of phasing plan, timing design and detector configuration. No analytical solution has been found to determine the optimal signal timing in actuated control. Traffic engineers continue to rely on computer simulation to generate signal timing plans (Simões et al. 2010).

Ribeiro and Simões (2010) have proposed a model to determine the optimal cycle length and green split allocation for an isolated signalized intersection regulated by traffic signals with a pre-timed control. The model in question has been formulated as a Mathematical Program with Equilibrium (or Complementarity) Constraints (MPEC) and solved by a Sequential Complementarity Algorithm (Slcp). The Slcp algorithm has always found the optimal solution requiring reduced computational time. However this formulation is not applicable to semi-actuated problems. Thus, a new formulation corresponding to this kind of control is presented in this paper and stated as a Mathematical Programming Problem with Equilibrium (or Complementarity) Constraints in the following manner:

$$
\text{(MPEC)} \quad \begin{array}{ll} \textit{Minimize} & c^T z + d^T y \\ \textit{subject to} & Ew = Mz + Ny + q \\ & z \geq 0,\, w \geq 0 \\ & y \in K_y \\ & w^T z = 0 \end{array}
$$

where $q \in R^p$, $c, z, w \in R^n$, $d, y \in R^m$, $M, E \in R^{p \times n}$, $N \in R^{p \times m}$ and $K_y = \{y \in R^m : Cy = b,\ y \geq 0\}$ with $C \in R^{l \times m}$ and $b \in R^l$.

A Sequential Linear Complementarity (Slcp) algorithm has been introduced by Júdice and Faustino (1992) to find a global minimum for a linear MPEC. This algorithm determines a sequence of stationary points of the MPEC with strictly decreasing values. The final stationary point of this sequence is proven to represent the global minimum of the MPEC. Computational experience reported in Júdice et al. (1992; 2002) indicates that the algorithm in question is rather efficient in finding a stationary point which is a global minimum of the MPEC. However, it is incapable of establishing that such a global minimum has indeed been achieved. In this paper the experiments carried out by this algorithm have indicated that, even in the specific case of a long period of time instants, the Slcp algorithm is capable of finding a global optimal solution for the model which is always the first stationary point attained in the sequence. It should also be noted that the computational effort required by this algorithm to obtain a solution is significantly small.

The organization of the paper is as follows. In Section 2, a description of the model is presented. Section 3 is devoted to the formulation of the model in question as a linear MPEC. Finally, computational experiments with regards to Slcp under a set of traffic problems and some conclusions are presented in the last section.

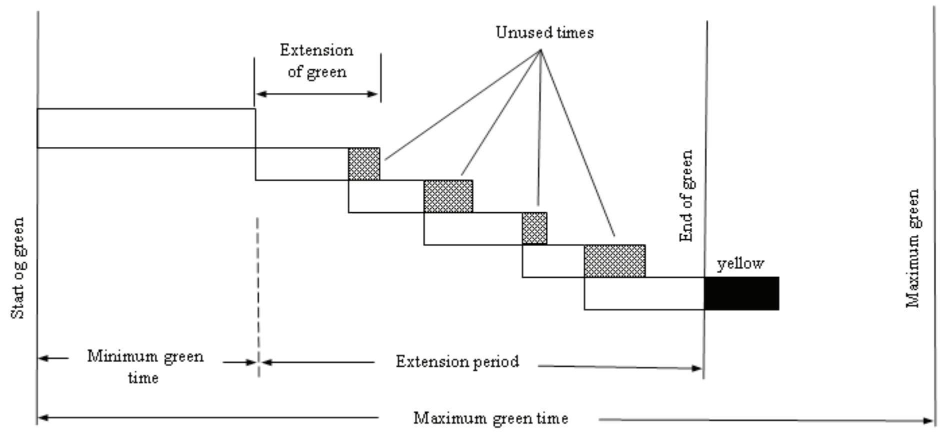

Figure 1 Extension sequence in vehicle-actuated control

2. Model description

The intersection presented below is constituted by a secondary street with an actuated control signal and a main street with no actuated control. This intersection is controlled by two phases (A and B). During phase A, the traffic signals T_1 and T_3 have a green light and the same occurs in phase B for T_2 and T_4. In both phases, the cycle has three states: green, yellow and red. The diagram of these phases has been illustrated in Figure 2 where we assume that there exist sensors near the traffic signals in each secondary street T_2 and T_4.

The arrival rate of vehicles in traffic stream S_i at time instant t is $\lambda_i(t)$ for i=1,2,3,4. When the traffic signal T_i is green, the departure rate in traffic stream S_i at time instant t is $\mu_i(t)$ and in the case of the traffic signal being yellow, the departure rate in traffic stream S_i at time t is $\kappa_i(t)$ for i=1,2,3,4. The design of timing plan is illustrated in Figure 3. The green time in traffic signals T_2 and T_4 is not constant, depends on the actuation of sensors. Let t_0, t_1, t_2,... be the time instants when a change in the traffic signals occurs.

We have assumed that the duration of the yellow time and the clearance time are fixed and have been set equal to d_Y and d_C, respectively.

The time instants when the traffic signals T_1 and T_3 initiate a green period and T_2 and T_4 begin a red period are t_0, t_2, t_4,.... The time instants when the traffic signals T_1 and T_3 initiate a red period and T_2 and T_4 begin a green period are t_1, t_3, t_5,.....

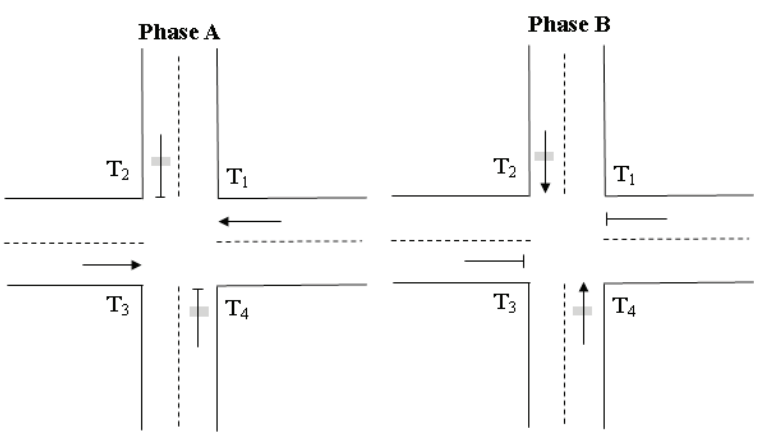

Figure 2 Diagram of phases

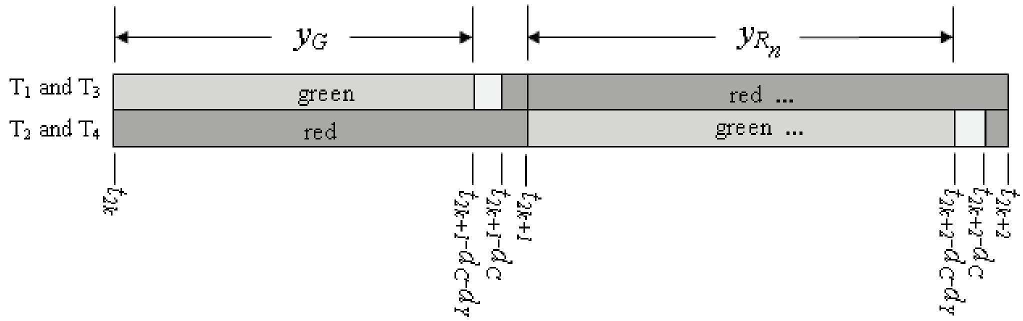

Figure 3 Diagram of signal timing

Thereby leading to $t_{2k+1} - t_{2k} = y_G + d_Y + d_C$ and $t_{2k+2} - t_{2k+1} = y_{Rn} + d_Y + d_C$, $k \in N_0$. Thus, y_G represents the green time in traffic signals T_1 and T_3 and y_{Rn} represents the red times T_1 and T_3 where n is the cycle number. d_Y and d_C represents the yellow time and clearance time, respectively. A cycle length is not constant and is equal to $y_G + y_{Rn} + 2d_Y + 2d_C$. Clearly, we should then have y_{Rn}, y_G, $d_Y \geq 0$. Furthermore, $\lambda_i(t)$, $\mu_i(t)$, $\kappa_i(t) \geq 0, \forall i,t$ and $t_k < t_{k+1}$, $\forall k$.

The queue length in the traffic stream S_i at instant time t, $L_i(t)$, is thus clearly equal to or greater than zero for all i and t. Whenever the traffic signal T_i is red, arrivals at traffic stream S_i are verified, which are characterized by the arrival rate function $\lambda_i(t)$. It should also be noted that there are no departures in this case. Alternatively, when the traffic signal T_i is green or yellow, there are both arrivals and departures at traffic stream S_i. In these cases, the net queue growth rate at the time instant t is $\lambda_i(t)$-$\mu_i(t)$ or $\lambda_i(t)$-$\kappa_i(t)$, respectively. Accordingly, for all streams, the evolution of the queue lengths can be stated as described by Ribeiro and Simões (2010).

Let us assume that, for each $i \in \{1,2,3,4\}$, $\bar{\lambda}_i$ is the average arrival rate and $\bar{\mu}_i$ and $\bar{\kappa}_i$ represent the average departure rates when the traffic signal is green or yellow, respectively. Let one further assume that $\lambda_i(t) = \bar{\lambda}_i$, $\mu_i(t) = \bar{\mu}_i$ and $\kappa_i(t) = \bar{\kappa}_i$ for all time instants t in addition to the queue not being empty, in which case the departure rates are equal to zero. Therefore arrivals and departures are considered to be deterministic.

3. Model Formulation

The following vectors are considered in a similar manner to that which has been described by Schutter and Moor (1998).

$$x_k = [L_1(t_k), L_2(t_k), L_3(t_k), L_4(t_k)]^T, \quad k \in N_0$$

$$b_1 = [\bar{\lambda}_1 - \bar{\mu}_1, \bar{\lambda}_2, \bar{\lambda}_3 - \bar{\mu}_3, \bar{\lambda}_4]^T$$

$$b_2 = [\bar{\lambda}_1, \bar{\lambda}_2 - \bar{\mu}_2, \bar{\lambda}_3, \bar{\lambda}_4 - \bar{\mu}_4]^T$$

$$b_3 = [(\bar{\lambda}_1 - \bar{\mu}_1)d_Y + \bar{\lambda}_1 d_C, \bar{\lambda}_2(d_Y + d_C), (\bar{\lambda}_3 - \bar{\mu}_3)d_Y + \bar{\lambda}_3 d_C, \bar{\lambda}_4(d_Y + d_C)]^T$$

$$b_4 = [\bar{\lambda}_1(d_Y + d_C), (\bar{\lambda}_2 - \bar{\kappa}_2)d_Y + \bar{\lambda}_2 d_C, \bar{\lambda}_3(d_Y + d_C), (\bar{\lambda}_4 - \bar{\kappa}_4)d_Y + \bar{\lambda}_4 d_C,]^T$$

$$b_5 = [\max\{(\bar{\lambda}_1 - \bar{\kappa}_1)d_Y + \bar{\lambda}_1 d_C, \bar{\lambda}_1 d_C\}, 0, \max\{(\bar{\lambda}_3 - \bar{\kappa}_3)d_Y + \bar{\lambda}_3 d_C, \bar{\lambda}_3 d_C\}, 0]^T$$

$$b_6 = [0, \max\{(\bar{\lambda}_2 - \bar{\kappa}_2)d_Y + \bar{\lambda}_2 d_C, \bar{\lambda}_2 d_C\}, 0, \max\{(\bar{\lambda}_4 - \bar{\kappa}_4)d_Y + \bar{\lambda}_4 d_C, \bar{\lambda}_4 d_C\}]^T$$

then

$$x_{2k+1} = \max\{x_{2k} + b_1 y_G + b_3, b_5\}$$

$$x_{2k+2} = \max\{x_{2k+1} + b_2 y_{R_n} + b_4, b_6\}$$

for $k \in N_0$ and n the cycle number.

In this study, the non-saturated intersections have been taken into consideration, which implies that the queue lengths may disappear when the traffic signal is green.

Let us assume that the arrival and departure rates have been previously determined. The model then seeks an optimal green split allocation for each phase. The objective function represents the total average waiting time experienced by vehicles in all queues:

$$J = \frac{1}{t_N - t_0} \sum_{i=1}^{4} \frac{1}{\lambda_i} \int_{t_0}^{t_N} L_i(t)dt \tag{1}$$

where N is the number of time instants and t_N - t_0 is the time interval considered, which it is not pre-timed.

One of the advantages of using criteria based on time averaged values is that the objective function has a finite value even if N or t_N tend to infinity, provided that the queue lengths remain finite.

Some additional conditions, such as the minimum and maximum durations for the red and green times or the maximum queue lengths, have been also added to the model since short cycles imply more stops and long cycles cause long delays, and as such, are unsuitable for the variations in the daily flow of traffic. This leads one to the following mathematical programming program:

$$\begin{array}{lll}
(P1) & \textit{Minimize} & J \\
 & \textit{subject to} & d_Y + d_C + g_{\min_A} \le y_{R_n} \le g_{\max_A} + d_Y + d_C \qquad (2) \\
 & & d_Y + d_C + g_{\min_B} \le y_G \le g_{\max_B} + d_Y + d_C \qquad (3) \\
 & & 0 \le x_k \le x_{\max} \qquad (4) \\
 & & x_{2k+1} = \max\{x_{2k} + b_1 y_G + b_3, b_5\} \qquad (5) \\
 & & x_{2k+2} = \max\{x_{2k+1} + b_2 y_{R_n} + b_4, b_6\} \qquad (6)
\end{array}$$

where $k \in N_0$, $g_{\min}$ and $g_{\max}$ are the minimum green time and maximum green time, respectively, in phase A(B) and $x_{\max}$ is the maximum queue length in each traffic stream.

However, for each index k, the nonlinear constraints (5) and (6) can be rewritten, respectively, as

$$\begin{cases} x_{2k+1} \ge x_{2k} + b_1 y_G + b_3 \\ x_{2k+1} \ge b_5 \\ (x_{2k+1} - x_{2k} - b_1 y_G - b_3)^T (x_{2k+1} - b_5) = 0 \end{cases}$$

and

$$\begin{cases} x_{2k+2} \ge x_{2k+1} + b_2 y_{R_n} + b_4 \\ x_{2k+2} \ge b_6 \\ (x_{2k+2} - x_{2k+1} - b_2 y_{R_n} - b_4)^T (x_{2k+2} - b_6) = 0 \end{cases}$$

and thus the objective function (1) can be replaced by

$$J = \sum_{i=1}^{4} \left(\frac{1}{2N} (x_0)_i + \sum_{k=1}^{N-1} \frac{1}{N} (x_k)_i + \frac{1}{2N} (x_N)_i \right)$$

If we consider,

$$z_{2k+1} = x_{2k+1} - b_5 \geq 0$$
$$z_{2k+2} = x_{2k+2} - b_6 \geq 0$$
$$w_{2k+1} = x_{2k+1} - x_{2k} - b_1 y_G - b_3 \geq 0$$
$$w_{2k+2} = x_{2k+2} - x_{2k+1} - b_2 y_{R_n} - b_4 \geq 0$$

then

$$\begin{cases} w_{2k+1} = z_{2k+1} - z_{2k} - b_1 y_G - b_3 + b_5 - b_6 \\ w_{2k+2} = z_{2k+2} - z_{2k+1} - b_2 y_{R_n} - b_4 - b_5 + b_6 \\ w \geq 0,\ z \geq 0 \\ w^T z = 0 \end{cases} \tag{7}$$

Thus, by taking into account the following constraints (2)-(3) and (7), problem (P1) can be reduced into the following linear Mathematical Programming Problem with Equilibrium (or Complementarity) Constraints:

$$\begin{array}{lll} \text{(MPEC)} & \textit{Minimize} & c^T z \\ & \textit{subject to} & w = Mz + Ny + q \\ & & l \leq y \leq u \\ & & z \geq 0,\ w \geq 0 \\ & & w^T z = 0 \end{array} \tag{8}$$

where $M \in R^{4N \times 4N}$, $N \in R^{4N \times (N/2+1)}$, $q \in R^{4N}$, $c \in R^{4N}$ and $l, u \in R^2$.

The Sequential Linear Complementarity (Slcp) algorithm is used with the goal of finding a global minimum for the MPEC. This algorithm computes a set of stationary points with strictly decreasing objective function values. It is also important to stress that the Slcp algorithm can be implemented by using an active-set code such as Minos (Murtagh and Saunders 1983). In fact, the Slcp algorithm only uses an enumerative method (Júdice et al. 2002) and a complementarity algorithm named by Caset (Judice et al. 2006) which are both implemented by using this type of methodology.

4. Computational Experiments

In this section, some of the more relevant computational experiments relating to the proposed traffic model, which exploits the MPEC formulation and uses the algorithm Slcp which has been carried out under the scope of this study, are reported. All the computations were performed on a Intel(R) Core(TM)2 Duo CPU 2.4GHz machine with 2 GB RAM

Table 1 presents parameters that distinguish each test problem *Prob*, namely, the four arrival rates $(\overline{\lambda}_i, i = 1,2,3,4)$, in addition to presenting the solutions attained by the Slcp for each test problem, in phase B (actuated signals):

GREEN: Average green times split
RED: Red time split
CYCLE: Average cycle lengths

In each of the test problems, a signalized intersection regulated by semi-actuated control has been considered with the following specifications:

$\overline{\mu}_i = 1800\ veh/h$ $\overline{\kappa}_i = 720 veh/h$ for i=1,2,3,4

$d_Y = 3s$ $d_C = 2s$

$g_{\min} = 7s$ $g_{\max A} = 40s$ $g_{\max B} = 30s$

$x_0 = 2\%\ \overline{\lambda}_k$ $x_0 = 2\%\ \overline{\lambda}_j$ for k=1,3, j=2,4

$x_{\max} = 25$ for i=1,2,3,4

Table 1 Experimental results

Prob	$\overline{\lambda}_1$ (veh/h)	$\overline{\lambda}_2$ (veh/h)	$\overline{\lambda}_3$ (veh/h)	$\overline{\lambda}_4$ (veh/h)	GREEN (s)	RED (s)	CYCLE (s)
P1	1100	330	1000	500	15.4	40.0	69.3
P2	950	285	850	425	14.9	40.0	67.8
P3	900	270	800	400	14.8	39.8	65.5
P4	850	298	750	450	14.4	38.6	68.0
P5	800	280	700	420	14.2	31.5	56.6
P6	750	263	650	390	14.2	27.5	49.7
P7	700	245	600	360	14.2	24.1	43.9
P8	650	228	550	330	14.5	25.9	45.2
P9	600	240	500	300	14.8	23.0	41.4
P10	1000	400	900	270	14.8	40.0	65.8
P11	900	360	800	240	14.6	38.7	62.2
P12	850	425	750	225	14.7	37.4	64.7
P13	900	540	800	480	24.0	40.0	74.6
P14	1000	600	900	270	23.3	40.0	73.3

Table 1 displays the average green times and red splits, in the actuated streams, for each test problem and the corresponding average cycle lengths. The resulting solutions lead to a minimum total delay of the intersection in question. The attained numerical results indicate that the average cycle length, in each test problem, reflects the correspondence with the traffic flow on each stream.

Figure 4 illustrates the evolution of the queue lengths formed on each of the four traffic streams for the test problem P1.

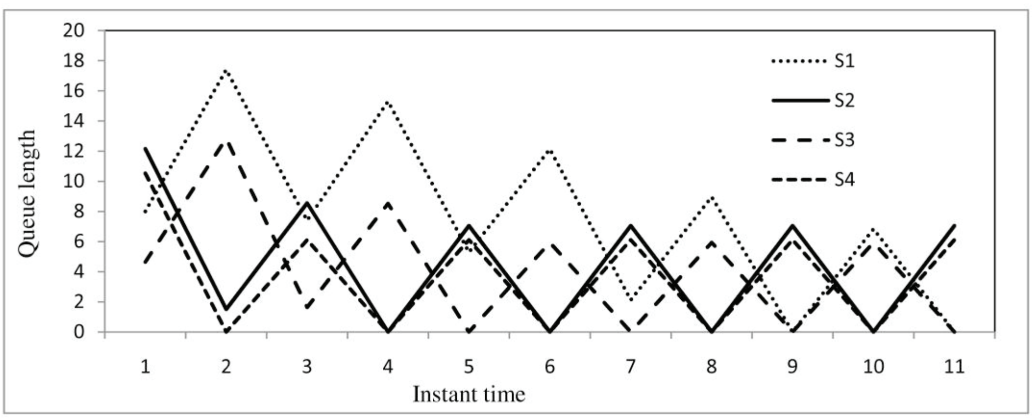

Figure 4 The evolution of queue lengths

One may observe (Figure 4) that the variability of the queue lengths in streams S_1 and S_3 verified in the initial time instants almost becomes constant throughout the final instants. However, in the actuated streams S_2 and S_4, from the beginning, the queue lengths are almost constant. This can be explained by the fact that the green times allocations are adapted to the number of vehicles detected by sensors. The instability at the outset is due to the initial conditions experienced which reflects the importance of the problem being solved for a large number of intervals, in order to eliminate this effect.

5. Conclusions

In this paper, the solution of a MPEC associated with traffic problems has been duly investigated. The formulation describes the evolution of the queue lengths at a signalised intersection regulated by semi-actuated control. Consequently, a Sequential Linear Complementarity (Slcp) algorithm to find a global minimum has been considered throughout this study.

The numerical results of various experiments reveal that it is possible to efficiently determine the average green time for the actuated streams and the average cycle length. The Slcp algorithm always finds the optimal solution in first iteration of the Slcp and requires reduced computational time. The solution attained by the present study corresponds to the minimum total delay of the intersection even when under saturation conditions. In addition, it meets the most current state of art solutions which are based on a queuing theory.

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